(pages 7, 36, 652), since there are quite a few mathematicians using the first alternative.

On the whole, the judgment from the first edition prevails that the book represents an excellent modern addition to the literature in numerical mathematics. The translation is also of high quality.

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1. W. Gautschi, Review **17**, Math. Comp. **28** (1974), 664–666; B. Parlett, Review **49**, *ibid.*, 1169; C.-E. Fröberg, Review **20**, *ibid.* **37** (1981), 600.

19[65N30, 65R20, 68Q25].—A. G. WERSCHULZ, *The Computational Complexity of Differential and Integral Equations: An Information-Based Approach*, Oxford Mathematical Monographs, Oxford Univ. Press, New York, 1991, x+331 pp., 24 cm. Price $55.00.

The focus of the book is the computational complexity of numerical methods (primarily finite element methods) for solving partial differential equations (PDEs). The book develops the so-called *information-based* approach which is part of a research program developed by J. F. Traub and coworkers [4]. There is a rare (for the mathematical sciences) controversy [1, 2, 4] regarding this line of research. Parlett [2] distinguishes between *numerical analysis* and what he refers to as the more challenging subject of *complexity theory.*

It is beyond the scope of this review to comment in depth on the controversy, but it is important for potential readers to know the extent of the controversy. The first section (2.1) after the Introduction in the article [2] by Parlett has the title "This is not complexity theory," referring to the information-based approach. This is a strong statement which (if accepted) would mean that the title of the book under review is misleading.

In justifying the proposition "This is not complexity theory," Parlett refers to complexity theory as a subject devoted to the *intrinsic difficulty* of a problem and criticizes the proponents of the information-based approach for restricting to a limited class of algorithms, rather than considering the intrinsic difficulty of the problem at hand. It is therefore worthwhile to consider these notions in some detail.

Complexity theory is always restricted to an explicit class of algorithms defined in a mathematical way. An excellent and accessible introduction to the complexity of basic arithmetic operations is [5]. Thus the comments of Parlett refer to the scope of the algorithms mentioned in §2.1 of [2], not the fundamental approach of limiting to a mathematically defined class of objects.

The term *intrinsic difficulty* should also be viewed in this context. For example, the prediction of whether a drop of honey will form and fall on your breakfast table before you get the spoon back to the honey jar requires the solution of an extremely complex free-boundary problem for a system of PDEs [3]. However, the intrinsic difficulty of the required experiment is minimal. Thus complexity theory typically works within a framework that would rule out a physical experiment being done to evaluate a function.

The book under review attempts to establish complexity theory for the solution of boundary value problems for PDEs, at least in the context of Galerkin methods. Some sort of restriction is required, as simple physical experiments can produce solutions of certain equations to remarkable accuracy. A major feature of complexity theory for PDEs is that the data and unknowns of a boundary value problem are infinite-dimensional in character, involving functions of continuum variables. The notion of *information*, a finite set of linear functionals of the data and unknowns, is introduced to circumvent this difficulty. For example, the value of the solution to Poisson's equation is sought at only N points, to an accuracy $\varepsilon > 0$, given the right-hand-side values only at a similar number of points.

The detailed results of the book require not only the full power of Sobolev spaces, but the definition of probability measures on them as well. It would be quite lengthy to describe a typical result, but the conclusions of one line of investigation shed light on a long-standing question of practical importance. In solving a problem with singular solutions, it has long been debated whether to use high-order or low-order methods. This book concludes that high-order methods are preferable, as the low-order methods would be potentially nonoptimal in complexity. Roughly speaking, the book points out that the order of the method should *exceed* the order of smoothness of the solution, otherwise all of the potential rate of convergence will not be realized. A lower-than-possible rate of convergence leads to a suboptimal algorithm, and there is no penalty (in terms of asymptotic complexity) for having an order of approximation greater than the level of smoothness of the solution.

The conclusion of another line of research is anticipated by Werschulz to be controversial, namely that adaptive methods are frequently no more efficient (in an asymptotic sense) than nonadaptive ones. Here the results are somewhat limited, and the results are predicated on various assumptions that may need further refinement. In particular, the results so far are restricted to linear problems. In any case, this intriguing opinion alone makes the book worth investigating in more detail.

Whether this book is, or is not, a part of complexity theory, it adds an interesting new dimension to the study of numerical methods for the solution of PDEs. Now that the numerical analysis of PDEs is well developed, it is certainly time to consider questions regarding the related complexity theory.

L. R. S.

1. M. W. Hirsch and R. S. Palais, *Editor's remarks*, Bull. Amer. Math. Soc. (N.S.) **26** (1992), 1–2.

2. B. N. Parlett, *Some basic information on information-based complexity theory*, Bull. Amer. Math. Soc. (N.S.) **26** (1992), 3–27.

3. W. G. Pritchard, L. R. Scott, and S. J. Tavener, *Numerical and asymptotic methods for certain viscous free-surface flows*, Philos. Trans. Roy. Soc. London Ser. A **340** (1992), 1–45.

4. J. F. Traub and W. Woźniakowski, *Perspectives in information-based complexity*, Bull. Amer. Math. Soc. (N.S.) **26** (1992), 29–52.

5. S. Winograd, *Arithmetic complexity of computations*, SIAM, Philadelphia, PA, 1980.